CHEM 332 – TAKE-HOME MIDTERM EXAMINATION FALL 2020

DATED: October 6, 2020
DUE: October 14, 2020

- Read the questions very carefully before answering.
- Do Problem 1a **OR** Problem 1b;
- Do Problem 2a **OR** Problem 2b.
- Do Problem 3.
- **One** set of solutions per group.
- A **single** group member should email me the answers copying all other group members.
- Indicate which group you belong to on your answer sheet.
- Show all steps in your calculations.
- 10 points set aside for creative solutions.

Problem 1a (35 points)
A particle of mass m is confined to a region $0 \leq x \leq$ a. The normalized wavefunction of the system is given by

$$\psi(x) = \sqrt{\frac{8}{5a}}\left[1 + cos\left(\frac{\pi x}{a}\right)\right] sin\left(\frac{\pi x}{a}\right)$$

(a) Find the average energy of the system.
(b) What is the probability of the finding the particle in the left half of the box?

Problem 1b (35 points)
For a particle constrained to move freely within the region $0 \leq x \leq$ a, the normalized wavefunction is given by

$$\psi(x) = \sqrt{\frac{630}{a^9}} x^2(a - x)^2$$

Calculate the variance in the energy $\sigma_E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$.

Problem 2a (10 points)

$$\hat{A} = \frac{d^2}{dx^2} - x$$

$$\hat{B} = \frac{d}{dx} + x^2$$

Evaluate $[\hat{A}, \hat{B}]$.

Problem 2b (10 points)

$$\hat{A} = \int_0^x dx'$$

$$\hat{B} = \frac{d}{dx}$$

Evaluate $[\hat{A}, \hat{B}]$.

Problem 3 (45 points)
A particle is constrained in one-dimension by a potential that is given by

$V(x) = V_0$ for $x < -\frac{a}{4}$
$V(x) = 0$ for $-\frac{a}{4} \leq x \leq +\frac{a}{4}$
$V(x) = V_0$ for $x > +\frac{a}{4}$

(a) Sketch the potential.
(b) Find the normalized wavefunctions and energies for an infinitely large V_0.
(c) In (b) find the expectation value for the position x.

Useful relations:

$$\int_0^a cos\left(\frac{n\pi x}{a}\right) cos\left(\frac{m\pi x}{a}\right) dx = \int_0^a sin\left(\frac{n\pi x}{a}\right) sin\left(\frac{m\pi x}{a}\right) dx = \frac{a}{2}\delta_{mn}$$

$\int_0^a sin\left(\frac{n\pi x}{a}\right) cos\left(\frac{m\pi x}{a}\right) dx = \int_0^a cos\left(\frac{n\pi x}{a}\right) sin\left(\frac{m\pi x}{a}\right) dx = 0$ for all m, n

$$\int_0^1 y^m(1-y)^n dy = \frac{m!\,n!}{(m+n+1)!}$$